3.1

14) $f(x) = \frac{1}{\sqrt{x}}$; $X_0 = 4$

find slope at x_0

$$\lim_{w \to x_0} \frac{f(w) - f(x_0)}{w - x_0} = \lim_{w \to x_0} \frac{\frac{1}{\sqrt{w}} - \frac{1}{\sqrt{x_0}}}{w - x_0}$$

$$= \lim_{w \to x_0} \frac{\frac{\sqrt{x_0} - \sqrt{w}}{\sqrt{w}\sqrt{x_0}}}{w - x_0} = \lim_{w \to x_0} \frac{\sqrt{x_0} - \sqrt{w}}{(w - x_0)\sqrt{w}\sqrt{x_0}}$$

$$= \lim_{w \to x_0} \frac{(\sqrt{x_0} - \sqrt{w})(\sqrt{x_0} + \sqrt{w})}{(w - x_0)\sqrt{w}\sqrt{x_0}(\sqrt{x_0} + \sqrt{w})} = \lim_{w \to x_0} \frac{(x_0 - w)}{(w - x_0)\sqrt{w}\sqrt{x_0}(\sqrt{x_0} + \sqrt{w})}$$

$$= \lim_{w \to x_0} \frac{-1}{\sqrt{w}\sqrt{x_0}(\sqrt{x_0} + \sqrt{w})} = \frac{-1}{\sqrt{x_0}\sqrt{x_0}(2\sqrt{x_0})}$$

$$= -\frac{1}{2} x_0^{-3/2}$$

① $$\frac{1}{\sqrt{w}} - \frac{1}{\sqrt{x_0}} = \frac{\sqrt{x_0} - \sqrt{w}}{\sqrt{w}\sqrt{x_0}}$$

$$\frac{1}{a} - \frac{1}{b} = \frac{b}{ab} - \frac{a}{ab} = \frac{b-a}{ab}$$

② $$\frac{\left(\frac{\sqrt{x_0} - \sqrt{w}}{\sqrt{w}\sqrt{x_0}}\right)}{w - x_0} = \frac{\sqrt{x_0} - \sqrt{w}}{(w - x_0)\sqrt{w}\sqrt{x_0}}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)\cdot\left(\frac{1}{c}\right)}{c\left(\frac{1}{c}\right)} = \frac{a}{bc}$$

③ $$\frac{(x_0 - w)}{(w - x_0)} = -1$$

$-b+a$

$$\frac{a-b}{b-a} = \frac{-(b-a)}{(b-a)} = -1$$

$$= \lim_{w \to x_0} \frac{-1}{\sqrt{w}\sqrt{x_0}(\sqrt{x_0}+\sqrt{w})} = \frac{-1}{\sqrt{x_0}\sqrt{x_0}(2\sqrt{x_0})}$$

$$= -\frac{1}{2}x_0^{-3/2}$$

$$\sqrt{a} = a^{1/2}$$

$$\frac{-1a^{-1/2}a^{-1/2}a^{-1/2}}{2}$$

$$-\frac{1}{2}a^{-3/2}$$

$$\frac{1}{b} = b^{-1}$$

So ..... $\frac{1}{\sqrt{a}} = a^{-\frac{1}{2}}$

$$a^b a^e a^n = a^{b+e+n}$$

$$-\frac{1}{2}(4)^{-3/2}$$

$$-\frac{1}{2}\left(\frac{1}{4}\right)^{3/2}$$

$$= -\frac{1}{2}\left[\left(\frac{1}{4}\right)^{\frac{1}{2}}\right]^{3} = -\frac{1}{2}\left(\frac{1}{2}\right)^{3}$$

$$= -\frac{1}{16}$$

3.2/14 $\sqrt{2x+1}$ * find $f'(x)$

$$f'(x) = \lim_{w \to x} \frac{f(w)-f(x)}{w-x} = \lim_{w \to x} \frac{\sqrt{2w+1}-\sqrt{2x+1}}{w-x}$$

$$= \lim_{w \to x} \frac{\left(\sqrt{2w+1}-\sqrt{2x+1}\right)}{w-x} \cdot \frac{\left(\sqrt{2w+1}+\sqrt{2x+1}\right)}{\left(\sqrt{2w+1}+\sqrt{2x+1}\right)}$$

$$= \lim_{w \to x} \frac{(2w+1)-(2x+1)}{(w-x)\left(\sqrt{2w+1}+\sqrt{2x+1}\right)} = \lim_{w \to x} \frac{2(w-x)}{(w-x)\left(\sqrt{2w+1}+\sqrt{2x+1}\right)}$$

$$= \lim_{w \to x} \frac{2}{\sqrt{2w+1}+\sqrt{2x+1}} = \frac{2}{\sqrt{2x+1}+\sqrt{2x+1}}$$

$$= \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

B. what about at $x=4$?

$$f'(4) = \left(\frac{1}{\sqrt{2x+1}}\right)\Bigg|_{x=4} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

eqn of tan. line:

slope AND (y-int or a point)

point: when $x=4$ what $f(4)$?

it is $\sqrt{2(4)+1} = 3$

so pt is $(4, 3)$

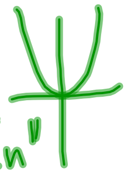
slope is $\frac{1}{3}$

so eqn: $y-3 = \frac{1}{3}(x-4)$

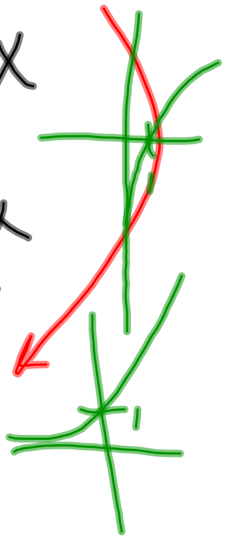
69 APMC/1 which of the following def^n "ODD"

a f^n f for which $f(-x) = -f(x)$

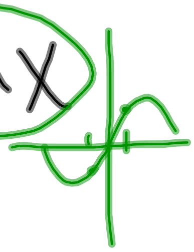
A) $f(x) = x^2$ ∴ "even"



D) $f(x) = \log x$

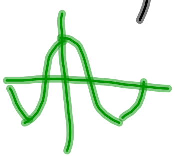


B) $f(x) = \sin x$ "odd"



E) $f(x) = e^x$

C) $f(x) = \cos x$ "even"



$f(-a) = -f(a)$

$f(-5) = -f(5)$

$f(-4) = -f(4)$

Q?] $a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^4-b^4=(a-b)(a^3+a^2b+ab^2+b^3)$

$a^5-b^5=(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$

$\vdots$

So... I should be able to make a "rule" for a^n-b^n

AND I can.

So... I should be able to make a rule for the DERIVATIVE of x^n

AND I can.

That's 3.3

3.1/12 $y = x^2+3x+2;\ x_0=2$

inst. rate of chge^{x_0} $= f'(x_0) = \lim_{w \to x_0} \dfrac{f(w)-f(x_0)}{w-x_0}$

$$= \lim_{w \to x_0} \frac{(w^2+3w+2)-(x_0^2+3x_0+2)}{w-x_0}$$

$$= \lim_{w \to x_0} \frac{w^2+3w+2-x_0^2-3x_0-2}{w-x_0}$$

$$= \lim_{w \to x_0} \frac{(w^2-x_0^2)+3(w-x_0)}{w-x_0}$$

$$= \lim_{w \to x_0} \left(\frac{\overset{(w-x_0)(w+x_0)}{w^2-x_0^2}}{w-x_0}\right)+3\left(\frac{w-x_0}{w-x_0}\right)$$

$$= \lim_{w \to x_0} (w+x_0)+3 = x_0+x_0+3 = 2x_0+3$$

b) $f'(2) = 2(2)+3 = 7$

I.E. the slope of the tangent line at $x=2$ is 7.

c) what is the eqn of the tangent line at $x=2$?

To write an eqn of a line, I ~~need~~: $y-y_0 = m(x-x_0)$ where (x_0, y_0) is a pt line goes through

$m = \dfrac{y-y_0}{x-x_0}$ ~~slope~~ (7)

pt $(2, f(2)) = (2, 12)$

EQN:

$y-12 = 7(x-2)$

$f(x) = x^2+3x+2$

$y-12 = 7x-14$
$\quad +12 \quad\quad +12$

$y = 7x-2$

2ND DRAW PRGM option 5 tangent

3.1/14

$$f(x)=\frac{1}{\sqrt{x}},\ x_0=1$$

$$f'(x_0)=\lim_{x_1\to x_0}\frac{f(x_1)-f(x_0)}{x_1-x_0}=\lim_{x_1\to x_0}\frac{\frac{1}{\sqrt{x_1}}-\frac{1}{\sqrt{x_0}}}{x_1-x_0}$$

ALG 2 ... ALG 1

$$=\lim_{x_1\to x_0}\frac{\sqrt{x_0}-\sqrt{x_1}}{\sqrt{x_1}\sqrt{x_0}(x_1-x_0)}\cdot\frac{(\sqrt{x_0}+\sqrt{x_1})}{(\sqrt{x_0}+\sqrt{x_1})}$$

$$=\lim_{x_1\to x_0}\frac{(x_0-x_1)}{\sqrt{x_1}\sqrt{x_0}(x_1-x_0)(\sqrt{x_0}+\sqrt{x_1})}$$

$$=\lim_{x_1\to x_0}\frac{-1}{\sqrt{x_1}\sqrt{x_0}(\sqrt{x_0}+\sqrt{x_1})}$$

$$=\frac{-1}{\sqrt{x_0}\sqrt{x_0}(\sqrt{x_0}+\sqrt{x_0})}=\frac{-1}{\sqrt{x_0}\sqrt{x_0}(2\sqrt{x_0})}$$

$$=-\frac{1}{2}x_0^{-3/2}$$

$$= \lim_{w \to x_0} \frac{-1}{\sqrt{w}\sqrt{x_0}\left(\sqrt{x_0}+\sqrt{w}\right)} = \frac{-1}{\sqrt{x_0}\sqrt{x_0}\left(2\sqrt{x_0}\right)}$$

girly ans $\boxed{\dfrac{-1}{2\sqrt{x_0^3}}}$

$$= -\frac{1}{2} x_0^{-3/2}$$

$$\sqrt{a} = a^{1/2}$$

$$\frac{-1 a^{-1/2} a^{-1/2} a^{-1/2}}{2}$$

$$-\frac{1}{2} a^{-3/2}$$

$$\frac{1}{b} = b^{-1}$$

So .... $\dfrac{1}{\sqrt{a}} = a^{-\frac{1}{2}}$

$$a^b a^e a^n = a^{b+e+n}$$

3.2 10) $f(x) = x^4$; $a = -2$

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x} = \lim_{w \to x} \frac{w^4 - x^4}{w - x}$$

$$= \lim_{w \to x} w^3 + w^2x + wx^2 + x^3 = x^3 + x^3 + x^3 + x^3$$

$$= 4x^3$$

see pg 12

$w^4 - x^4 \neq (w-x)^4$

Multiply out $(w-x)^4$

$= (w-x)(w-x)(w-x)(w-x)$

10 cont...

$f(x) = x^4$; $x_0 = -2$

$f'(x) = 4x^3$

b.... what is slope @ $x = -2$?

$f'(-2) = 4(-2)^3 = -32$

so what is eqn of tan line @ $x = -2$?

slope: -32

pt $(-2, f(-2)) = (-2, (-2)^4) = (-2, 16)$

line:

$y - 16 = -32(x - (-2))$

$y - y_0 = m(x - x_0)$

$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$\cdot\, y_1 - y_0 \quad \cdot\, y_1 - y_0$

$m(x_1 - x_0) = f(x_1) - f(x_0)$

$$x^2-y^2=(x-y)(x+y)$$

$$x^3-y^3=(x-y)(x^2+xy+y^2)$$

$$x^4-y^4=(x-y)(x^3+x^2y+xy^2+y^3)$$

$$x^5-y^5=(x-y)(x^4+x^3y+x^2y^2+xy^3+y^4)$$

Maybe... general rule for $\frac{x^n-y^n}{x-y}$

And.... THERE IS!

But wait!! maybe ... general rule for

derivative of x^n... AND...

THERE IS!!

that's 3.3...

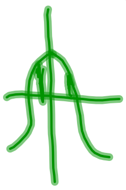
69 APMC#1) which of the following def^n a f^n f for which $f(-x) = -f(x)$

odd

A) $f(x) = x^2$

even

B) $f(x) = \sin x$

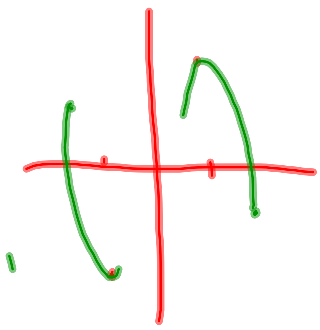


C) $f(x) = \cos x$

even

D) $f(x) = \log x$

E) $f(x) = e^x$



$f(-a) = -f(a)$

$f(-2) = -f(2)$

$f(3) = -f(-3)$